$\boxed{n \text{ cavie}}$

$\rightarrow \begin{cases} 1 \rightarrow \text{reazione} \\ 0 \rightarrow \text{no reazione} \end{cases}$

oppure

$1 \rightarrow$ no reazione

$0 \rightarrow$ reazione

$1 \rightarrow$ successo

$0 \rightarrow$ insuccesso

$\rightarrow p$ = prob. di reazione

Indipendenza

Formula di Bayes

$A_1 \longrightarrow 40\%$
- L 52%
- B 48%

$A_2 \longrightarrow 60\%$
- L 37%
- B 63%

Si riceve un segnale B. Calc. la prob. che il segnale provenga da A_1

$A_1 = \{$ il segnale viene da $A_1 \}$

$A_2 = \{$ " " " " $A_2 \}$

$B = \{$ il segnale è breve $\}$

$L = \{$ " " " lungo $\}$

$P(A_1) = 0.4$

$P(A_2) = 0.6$

$P(A_1 | B) = ?$

$P(L | A_1) = 0.52$

$P(B | A_1) = 0.48$

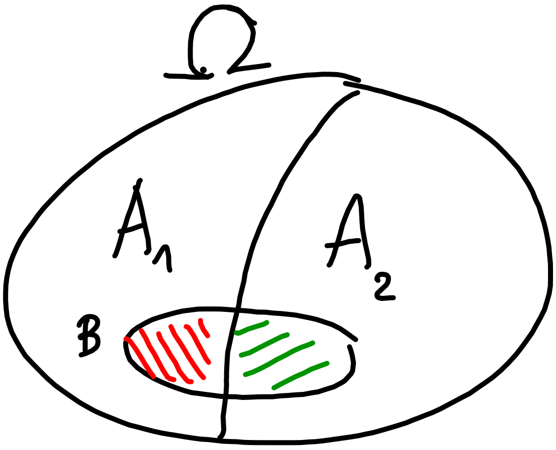
$P(L | A_2) = 0.37$

$P(B | A_2) = 0.63$

$$P(A_1 | B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{0.48 \times 0.4}{?}$$

$$\cancel{P(A_1)}\frac{P(A_1 \cap B)}{\cancel{P(A_1)}} = \underbrace{P(B|A_1)} \underbrace{P(A_1)} =$$

$$= 0.48 \times 0.4$$



- $A_1 \cup A_2 = \Omega$
- $\underline{A_1 \cap A_2} = \emptyset$

$$B = B \cap \Omega = B \cap (A_1 \cup A_2) =$$

$$= (\underline{B \cap \widehat{A}}_1) \cup (\underline{B \cap \widehat{A}}_2)$$

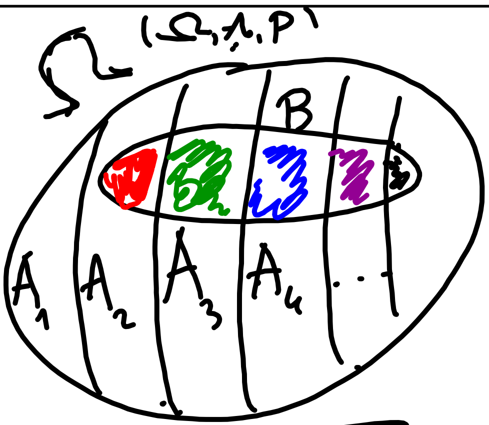
$$P(B) = P(B \cap A_1) \overset{*}{+} P(B \cap A_2) =$$

$$= P(B|A_1)\,P(A_1) + P(B|A_2)\,P(A_2)$$

$$P(B) = 0,48 \times 0,4 + 0,63 \times 0,6$$

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2)}$$

$$= \frac{0,48 \times 0,4}{0.48 \times 0,4 + 0,63 \times 0,6} = \ldots$$

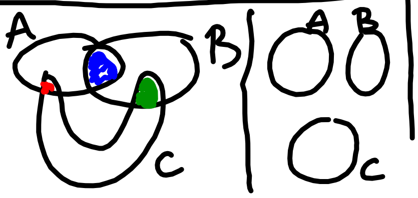


$(A_k)_{k=1,\dots n}$ è una partizione di Ω

1) $A_k \in \mathcal{A} \quad \forall k$

2) $\bigcup_{k=1}^{n} A_k = \Omega$

3) $A_h \cap A_k = \emptyset \quad \forall h \neq k$



$P(A_k) \quad \forall k = 1, \dots, n$

$(P(A_k) > 0 \quad \forall k)$

$P(B|A_k) \quad \forall k = 1, \dots n$

$$P(A_h|B) = \frac{P(A_k \cap B)}{P(B)}$$

$$P(A_h \cap B) = P(B|A_k)\,P(A_k)$$

$$P(B) = \sum_{k=1}^{n} P(B \cap A_k)$$

Variabili aleatorie

sequenze di lunghezza 5, simboli 0,1

$(0,0,0,1,1) \longrightarrow 9+4=13\ \mu sec$

$(0\,0\,1\,1\,0)$

$1 \longrightarrow 2\ \mu sec$

$0 \longrightarrow 3\ \mu sec$

$$\Omega = \{0,1\}^5$$

p = prob che venga il simbolo 1

$$P(\{\omega\}) = p^{\sum_{i=1}^{5}\omega_i}(1-p)^{5-\sum_{i=1}^{5}\omega_i}$$

$$(\Omega, \mathcal{A}, P)$$

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)$$

$$\omega = (0,0,0,1,1) \longrightarrow 13$$

$$\bar{\omega} = (1,1,0,1,1) \longrightarrow 11$$

$$\forall \omega \in \Omega$$

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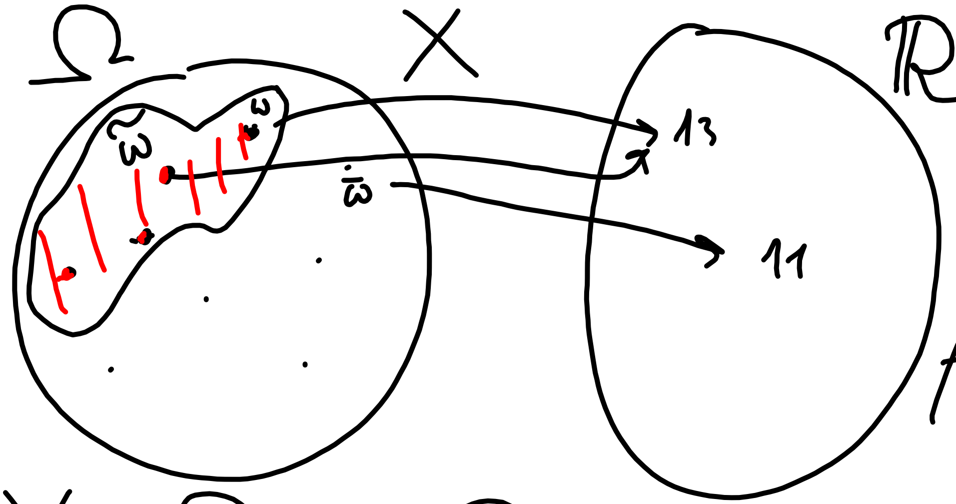


$X^{-1}(\{13\})$

$\mathcal{A}_0 = \mathcal{P}(\Omega)$

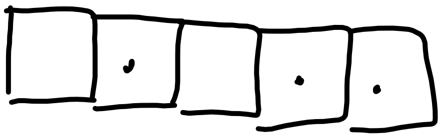
$X : \Omega \longrightarrow \mathbb{R}$ = tempo necessario

$(0,0,0,1,1)$ $(0\,0,1,1,0,1)$

$p^2(1-p)^3$ $(1\,0\,1\,0\,0)$

$\vdots$

$(1,1,0,0,0)$



$10\,p^2(1-p)^3$

$$\binom{5}{3} = \frac{5!}{3!\,2!} = \frac{4\cdot 5}{2!} = 10$$

Sia $(\Omega, \mathcal{A}, P)$ uno spazio di probabilità e sia $X : \Omega \to \mathbb{R}$ una funzione.

<u>Definizione</u> X si chiama <u>variabile aleatoria</u> (casuale, random variable) <u>v.a.</u> (r.v.) se $\forall t \in \mathbb{R}$

$$\{\omega \in \Omega : X(\omega) \leq t\} \in \mathcal{A}$$

$$= \{\omega \in \Omega : X(\omega) \in (-\infty, t]\}$$

$$= \{X \in (-\infty, t]\}$$

set 29-17:21

$$\{\omega \in \Omega : \; a < X(\omega) \le b \} =$$

$$= \{\omega \in \Omega : X(\omega) \in (a,b] \} =$$

$$= X^{-1}\big((a,b]\big) \qquad a < b$$

$$\{\omega \in \Omega : \; a < X(\omega) \le b \} =$$

$$= \{\omega \in \Omega : X(\omega) \le b \} \cap \{X(\omega) > a\} =$$

$$= \overbrace{\{\omega : X(\omega) \le b\}}^{\in \mathcal{A}} \cap \overbrace{\{X(\omega) \le a\}}^{\in \mathcal{A}}{}^{c} \in \mathcal{A}$$

$$P(\underbrace{\{\omega \in \Omega : X(\omega) = 13\}}) = P(\underline{X = 13})$$

$$P(\{\omega \in \Omega : X(\omega) \leq 13\}) =$$

$$= P(X \leq 13)$$

$$P(X > 11)$$

set 29-17:03

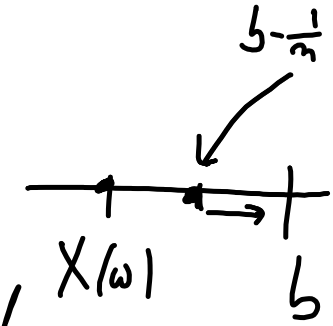
$$\{\omega : X(\omega) > a\} =$$

$$= \{\omega : X(\omega) \le a\}^c$$

$$\{\omega : X(\omega) < b\}$$

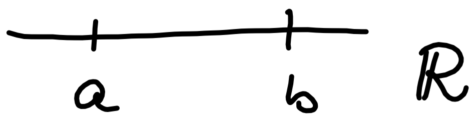
$$\{\omega : X(\omega) < b\} =$$

$$= \bigcup_{n\in\mathbb{N}} \underbrace{\{\omega: X(\omega) \le b - \tfrac{1}{n}\}}_{\in \mathcal{A}_b} \in \mathcal{A}_b$$



$\boxed{\sigma}$- algebra

$$\{a < X < b\} = \qquad a < x < b$$



$$= \underbrace{\{X < b\}}_{\in \mathcal{A}_0} \cap \underbrace{\{X \leq a\}^c}_{\in \mathcal{A}_0}$$

$$\{ a \le X < b \} =$$

$$= \{ \underline{X < b} \} \cap \{ X < a \}^c$$

$$\{ X = a \} = \{ \underbrace{X \le a}_{\in \mathcal{F}} \} \cap \{ \overbrace{X < a}^{\in \mathcal{F}} \}^c$$

$\{X \in I\}$

$P(X \in I)$ al variare di I

Definizione. La funzione

$I \longmapsto P(X \in I)$

si chiama legge di X

Variabili ~~ale~~ aleatorie discrete

I intervalli in $\mathbb{R}$

$[a, b] \quad (a, b) \quad [a, b)$

$(a, b] \quad [a, a] = \{a\}$

$(-\infty, a] \quad (-\infty, a) \quad [a, +\infty)$

$(a, +\infty) \quad (-\infty, +\infty)$

$$\{\omega \in \Omega : X(\omega) \leq 13\}$$
$$= \{\omega \in \Omega : X(\omega) \in (-\infty, 13]\}$$
$$\{\omega \in \Omega : X(\omega) > 11\} =$$
$$= \{\omega \in \Omega : X(\omega) \in (11, +\infty)\}$$

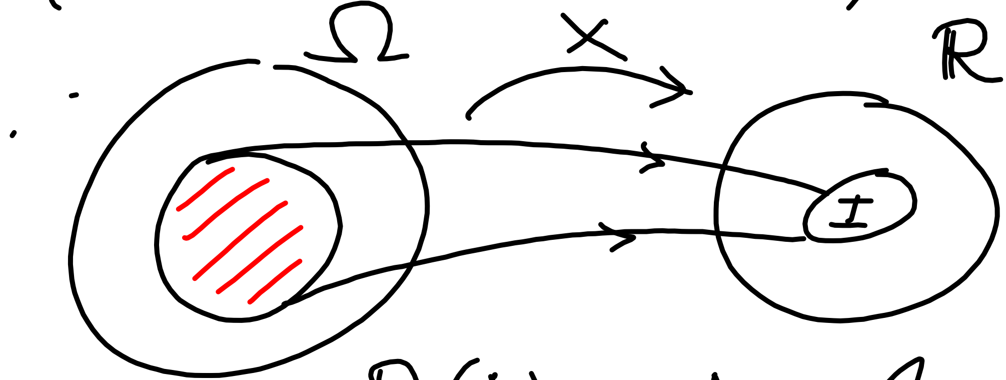
$\{\omega \in \Omega : 2 \leq X(\omega) < 5\} =$

$= \{\omega \in \Omega : X(\omega) \in [2,5)\}$

$= \{X \in [2,5)\}$

$P(X \in [2,5))$ $\{\omega \in$

$$A = \{\omega \in \Omega : X(\omega) \in I\} = X^{-1}(I)$$



$P(A) \qquad A \in \mathcal{A}$

set 29-17:14

$$B = B \cap \Omega = B \cap \left(\bigcup_{k=1}^{n} A_k \right) =$$

$$= \bigcup_{k=1}^{n} \underbrace{(B \cap A_k)}$$

$$P(B) = \sum_{k=1}^{n} P(B \cap A_k) = \sum_{k=1}^{n} P(B|A_k) P(A_k)$$

$$P(A_h|B) = \frac{P(B|A_h)\,\underline{\underline{P(A_h)}}}{\sum_{k=1}^{n} \underline{P(B|A_k)}\,\underline{\underline{P(A_k)}}}$$

prob a posteriori

prob. a priori

Formula di Bayes